MODULE-I

D.C NETWORKS

1.1 Kirchoff's Laws:-

1.1.1. Kirchoff's current law or point law (KCL)

Statement:- In any electrical network, the algebraic sum of the currents meeting at a point is zero.

$\Sigma I = 0$ ....at a junction or node

Assumption:- Incoming current = positive

Outgoing current = negative

1.1.2. Kirchoff's voltage law or mesh law (KVL)

Statement:- The algebraic sum of the products of currents and resistances in each of the conductors in any closed path (or mesh) in a network plus the algebraic sum of the emfs in that path is zero.

$\Sigma IR + \Sigma emf = 0$ ....round the mesh

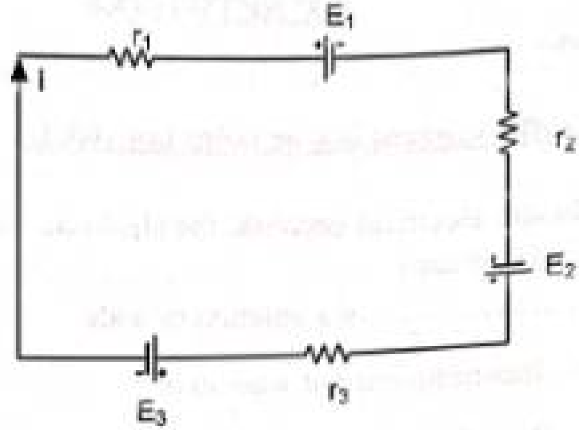
Assumption:- i) Rise in voltage (If we go from negative terminal of the battery to positive terminal) = positive

ii) Fall in voltage (If we go from positive terminal of the battery to negative terminal) = negative

iii) If we go through the resistor in the same direction as current then there is a fall in potential. Hence this voltage is taken as negative.

iv)If we go through the resistor against the direction of current then there is a rise in potential. Hence this voltage drop is taken as positive.

Example:- Write the loop equation for the given circuit below **(Supplementary exam 2004)**



Solution: Apply KVL to the loop,

$$-ir_1 - E_1 - ir_2 + E_2 - ir_3 - E_3 = 0$$

$$\Rightarrow E_1 - E_2 + E_3 = -ir_1 - ir_2 - ir_3$$

$$\Rightarrow E_1 - E_2 + E_3 = -i\left(r_1 + r_2 + r_3\right)$$

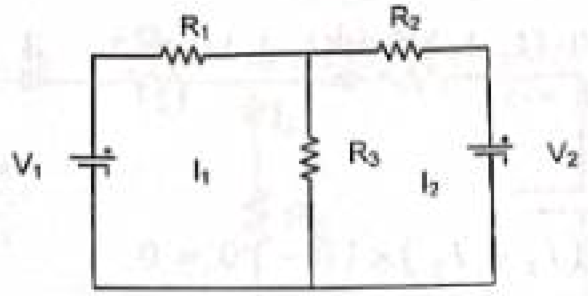
1.2. MAXWELL'S LOOP CURRENT METHOD (MESH ANALYSIS)

Statement:- This method determines branch currents and voltages across the elements of a network. The following process is followed in this method:-

- Here, instead of taking branch currents (as in Kirchoff's law) loop currents are taken which are assumed to flow in the clockwise direction.
- Branch currents can be found in terms of loop currents
- Sign conventions for the IR drops and battery emfs are the same as for Kirchoff's law.
- This method is easier if all the sources are given as voltage sources. If there is a current source present in a network then convert it into equivalent voltage source.

Explanation:-

Consider a network as shown in Fig. below. It contains two meshes. Let I_1 and I_2 are the mesh currents of two meshes directed in clockwise.



Apply KVL to mesh-1,

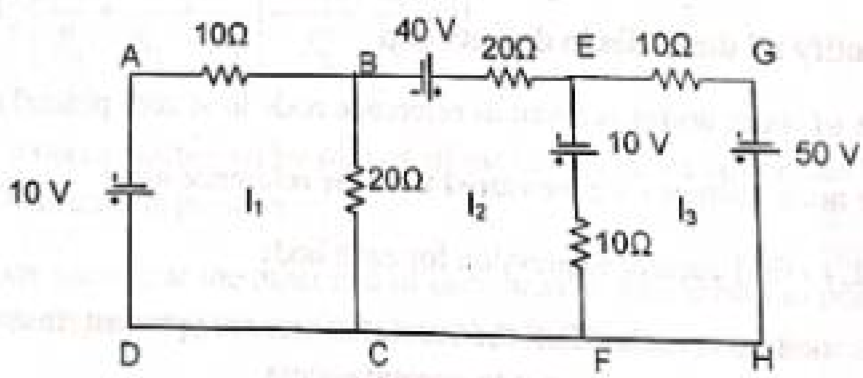
$$V_1 - I_1 R_1 - (I_1 - I_2) R_3 = 0$$

Apply KVL to mesh-2,

$$-I_2 R_2 - V_2 - (I_2 - I_1) R_3 = 0$$

When we consider mesh-1, the current I_1 is greater than I_2. So, current through R_3 is I_1-I_2. Similarly, when we consider mesh-2, the current I_2 is greater than I_1. So, current through R_3 is $I_2 - I_1$.

Example: Find I_1, I_2 and I_3 in the network shown in Fig below using loop current method



Solution:- For mesh ABCDA,

$$-I_1 \times 10-(I_1-I_2)\times 20-10=0$$
$$\Rightarrow 3I_1-2I_2=-1 \qquad (1)$$

For mesh BEFCB,

$$40-I_2\times 20+10-(I_2-I_3)\times 10-(I_2-I_1)\times 20=0$$
$$\Rightarrow 2I_1-5I_2+I_3=-5 \qquad (2)$$

For mesh EGHFE,

$$-10I_3+50-(I_3-I_2)\times 10-10=0$$
$$\Rightarrow I_2-2I_3=-4 \qquad (3)$$

Equation (2) x 2 + Equation (3)

$$4I_1-9I_2=-14 \qquad (4)$$

Solving eq^n (1) & eq^n (4)

$I_1 = 1$ A, $I_2 = 2$ A, $I_3 = 3$ A

1.3. NODAL ANALYSIS

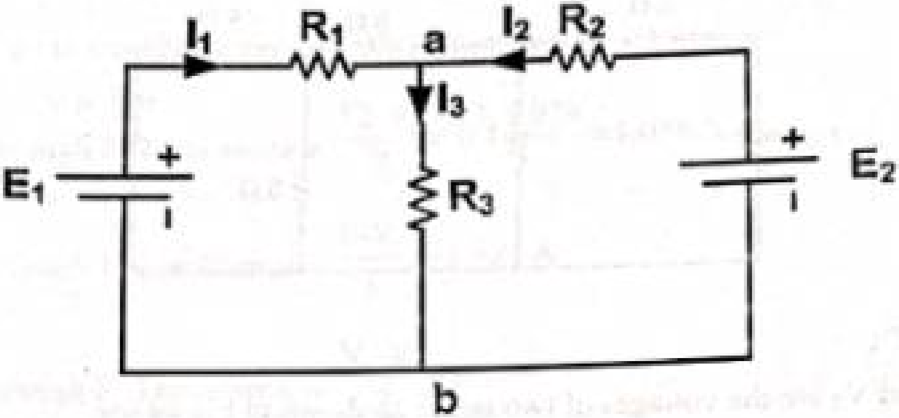
Statement:- This method determines branch currents in the circuit and also voltages at individual nodes.

The following steps are adopted in this method:-

- Identify all the nodes in the network.
- One of these nodes is taken as reference node in at zero potential
- The node voltages are measured w.r.t the reference node
- KCL to find current expression for each node
- This method is easier if all the current sources are present. If any voltage source is present, convert it to current source

- The number of simultaneous equations to be solved becomes (n-1) where 'n' is the number of independent nodes.

Explanation:-



At node 'a' $I_1+I_2 = I_3$

By ohms law, $I_1 = \frac{E_1 - V_a}{R_1}, I_2 = \frac{E_2 - V_a}{R_2}, I_3 = \frac{V_a}{R_3}$

Therefore,

$$V_a\left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right]-\frac{E_1}{R_1}-\frac{E_2}{R_2}=0$$

or,

$$V_a\left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right]-\frac{E_1}{R_1}-\frac{E_2}{R_2}=0$$

or,

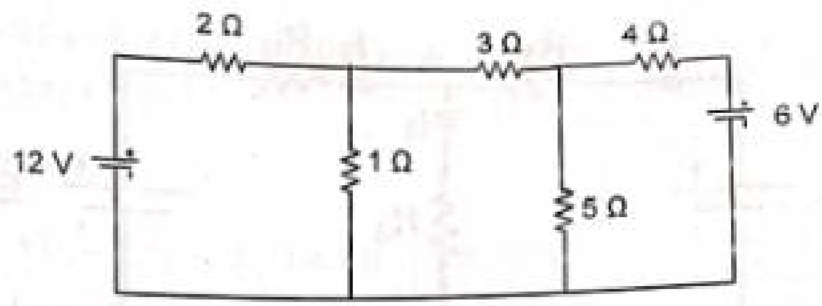
$$V_a\left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right]-\frac{E_1}{R_1}-\frac{E_2}{R_2}=0$$

Hence,

- Node voltage multiplied by sum of all the conductance connected to this node. This term is positive
- The node voltage at the other end of each branch (connected to this node multiplied by conductance of this branch). This term is negative.

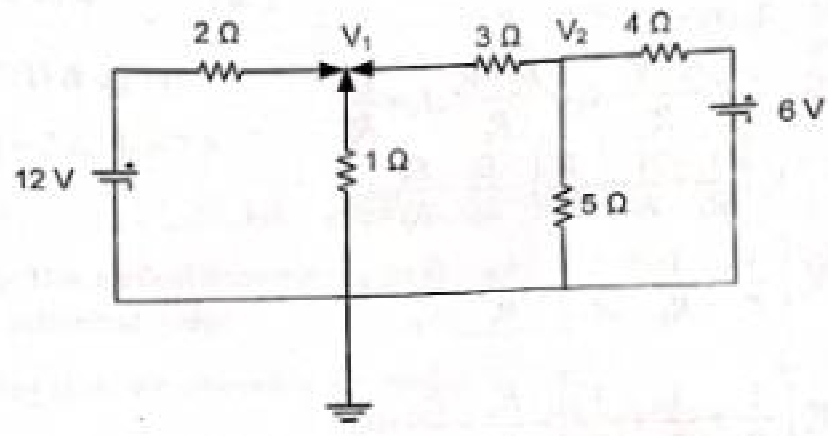
Example:- Use nodal analysis to find currents in the different branches of the circuit shown below.

(Supplementary Exam July- 2004)



Solution:-

Let V_1 and V_2 are the voltages of two nodes as shown in Fig below



Applying KCL to node-1, we get

$$\frac{12 - V_1}{2} + \frac{0 - V_1}{1} + \frac{V_2 - V_1}{3} = 0$$

$$\Rightarrow 36 - 3V_1 - 6V_1 + 2V_2 - 2V_1 = 0$$

$$\Rightarrow -11V_1 + 2V_2 = 36 \text{..............(1)}$$

Again applying KCL to node-2, we get:-

$$\frac{V_1 - V_2}{3} + \frac{0 - V_2}{5} + \frac{6 - V_2}{4} = 0$$

$$\Rightarrow 20V_1 - 47V_2 + 90 = 0$$

$$\Rightarrow 20V_1 - 47V_2 = -90 \text{..............}(2)$$

Solving Eq (1) and (2) we get $V_1 = 3.924$ Volt and $V_2 = 3.584$ volt

Current through 2 Ω resistance = $\frac{12-V_1}{2} = \frac{12-3.924}{2} = 4.038$ A

Current through 1 Ω resistance = $\frac{0-V_1}{1} = -3.924$ A

Current through 3 Ω resistance = $\frac{V_1-V_2}{3} = 0.1133$ A

Current through 5 Ω resistance = $\frac{0-V_2}{5} = -0.7168$ A

Current through 4 Ω resistance = $\frac{6-V_2}{4} = 0.604$ A

As currents through 1Ω and 5Ω are negative, so actually their directions are opposite to the assumptions.

1.4. STAR-DELTA CONVERSION

Need:- Complicated networks can be simplified by successively replacing delta mesh to star equivalent system and vice-versa.

In delta network, three resistors are connected in delta fashion (Δ) and in star network three resistors are connected in wye (Y) fashion.

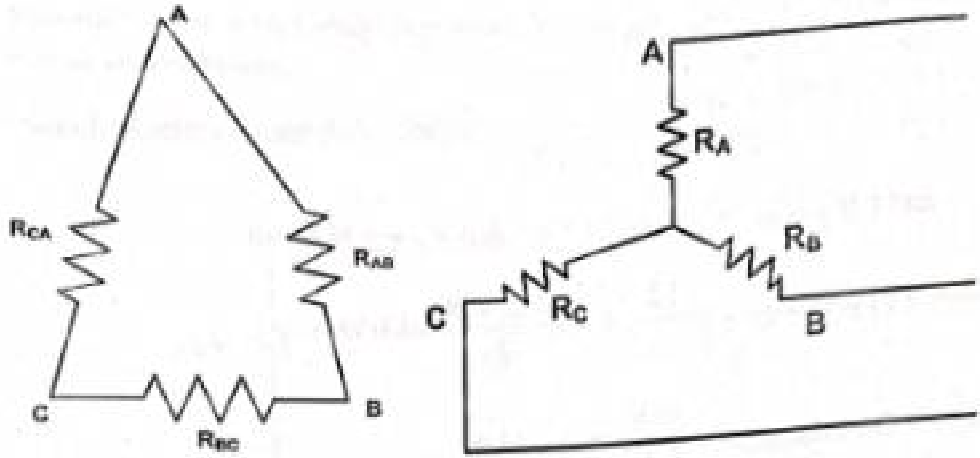


Fig. 1.4.1.

a) Delta connection b) Star connection

1.4.1. **Delta to Star Conversion**:- From Fig. 1.4.1 (a), Δ : Between A & B, there are two parallel path.

Resistance between terminal A & B = $\frac{R_{AB}(R_{BC}+R_{CA})}{R_{AB}+R_{BC}+R_{CA}}$

From Fig. 1.4.1 (b), STAR: Between A & B two series resistances are there $R_A + R_B$. So, terminal resistances have to be the same.

$$R_A + R_B = \frac{R_{AB}(R_{BC}+R_{CA})}{R_{AB}+R_{BC}+R_{CA}} \text{.................(1)}$$

$$R_B + R_C = \frac{R_{BC}(R_{CA}+R_{AB})}{R_{AB}+R_{BC}+R_{CA}} \text{.................(2)}$$

$$R_C + R_A = \frac{R_{CA}(R_{AB}+R_{BC})}{R_{AB}+R_{BC}+R_{CA}} \text{.................(3)}$$

Eq {(1)-(2)}+(3) & Solving,-

$$R_A = \frac{R_{AB}\times R_{CA}}{R_{AB}+R_{BC}+R_{CA}} \text{.................(4)}$$

$$R_B = \frac{R_{AB}\times R_{BC}}{R_{AB}+R_{BC}+R_{CA}} \text{.................(5)}$$

$$R_C = \frac{R_{CA} \times R_{BC}}{R_{AB} + R_{BC} + R_{CA}} \quad \ldots\ldots\ldots\ldots\ldots\ldots(6)$$

Easy way to remember:-

$$Any\ arm\ of\ star\ connection = \frac{Product\ of\ two\ adjacent\ arms\ of\ delta}{sum\ of\ arms\ of\ delta}$$

1.4.2. Star to Delta conversion

Eq [(1) X (2)]+(2) X (3)+ (3) X (1) & Simplifying.-

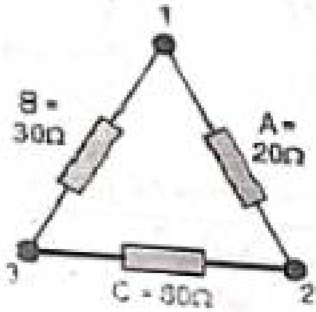
$$R_{AB} = \frac{R_A R_B + R_B R_C + R_C R_A}{R_C} = R_A + R_B + \frac{R_A R_B}{R_C}$$

$$R_{BC} = R_B + R_C + \frac{R_B R_C}{R_A}$$

$$R_{CA} = R_C + R_A + \frac{R_C R_A}{R_B}$$

Easy way to remember:- *Resistance between two terminals of delta = sum of star resistance connected to those terminals + product of the same to resistance divided by the third resistance.*

Example(delta to star):- Convert the following Delta Resistive Network into an equivalent Star Network.



$$Q = \frac{AC}{A+B+C} = \frac{20\times 80}{130} = 12.31\Omega$$

$$P = \frac{AB}{A+B+C} = \frac{20\times 30}{130} = 4.61\Omega$$

$$R = \frac{BC}{A+B+C} = \frac{30\times 80}{130} = 18.46\Omega$$

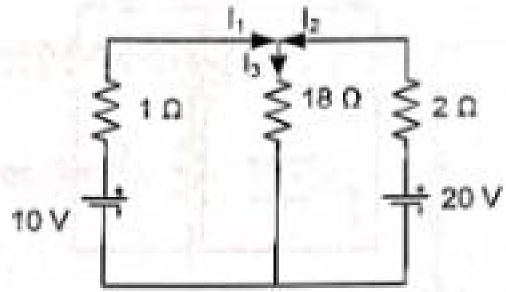
NETWORK THEOREMS

- SUPERPOSITION THEOREM
- THEVENIN'S THEOREM
- NORTON'S THEOREM
- MAXIMUM POWER TRANSFER THEOREM

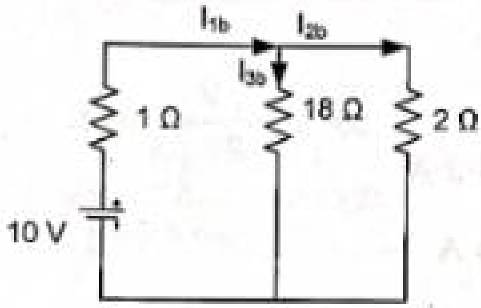
1. Superposition theorem

Statement:- In a network of linear resistances containing more than one generator (or source of emf), the current which flows at any point is the sum of all the currents which would flow at that point if each generator were considered separately and all the other generators replaced for the time being by resistances equal to their internal resistance.

Example:- By means of superposition theorem, calculate the currents in the network shown.



Step 1. <u>Considering 10 V battery</u>



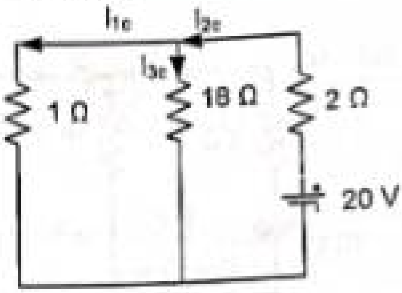
$$R_{eq} = \frac{2 \times 18}{2 + 18} + 1 = 2.8\,\Omega$$

$$I_{1b} = \frac{10}{2.8} = 3.57\ \text{A}$$

$$I_{2b} = 3.57 \times \frac{18}{20} = 3.21\ \text{A}$$

$$I_{3b} = I_{1b} - I_{2b} = 0.36\ \text{A}$$

Step 2. Considering 20 V battery



$$R_{eq} = \frac{1 \times 18}{1 + 18} + 2 = 2.95\,\Omega$$

$$I_{2c} = \frac{20}{2.95} = 6.78\,A$$

$$I_{1c} = 6.78 \times \frac{18}{19} = 6.42\,A$$

$$I_{3b} = I_{2c} - I_{1c} = 0.36\,A$$

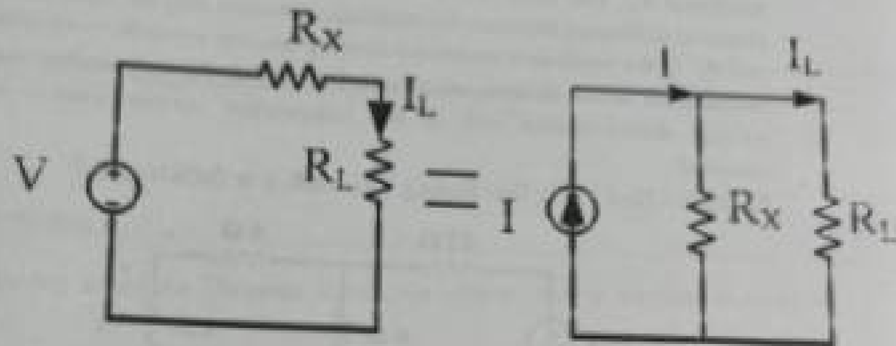
Step 3. Results

$$I_1 = I_{1b} - I_{1c} = 3.57 - 6.42 = -2.85A$$
$$I_2 = I_{2c} - I_{2b} = 6.78 - 3.21 = 3.57A$$
$$I_3 = I_{3b} + I_{3c} = 0.36 + 0.36 = 0.72A$$

2. **SOURCE CONVERSION**:-

Statement: A voltage source (V) with a series resistance (R) can b converted to a current source (I=V/R) with a parallel resistance (R) and vic versa.

Proof:-



$$I_L = \frac{V}{R_X + R_L} \qquad (1)$$

$$I_L = I\frac{R_X}{R_X + R_L} \qquad (2)$$

From Eq. (1) & (2)

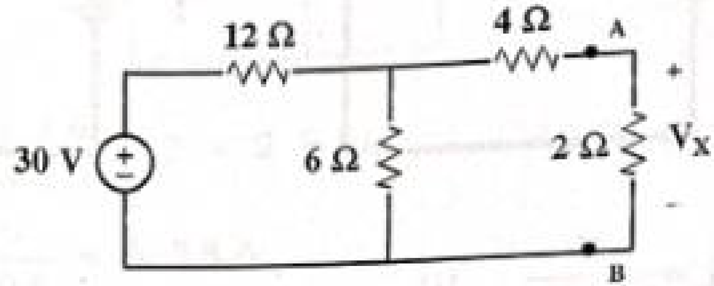
$$V = IR_X \qquad (3)$$

- **STATEMENT**: The two circuits are said to be electrically equivalent if they supply equal load currents with the same resistance connected across their terminals.
- voltage source having a voltage V and source resistance Rx can be replaced by I(= V/Rx) and a source resistance Rx in parallel with current source.
- Current source I and source resistance Rx can be replaced by a voltage source V (=IRx) and a source resistance Rx in series with V.

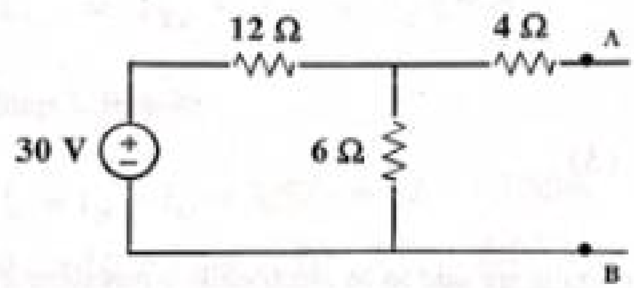
3. **THEVENIN'S THEOREM**:-

Statement:- Any pair of terminals *AB* of a linear active network may be replaced by an equivalent voltage source in series with an equivalent resistance R_{th}. The value of V_{th} (called the Thevenin's voltage) is equal to potential difference between the terminals AB when they are open circuited and R_{th} is the equivalent resistance looking into the network at AB with the independent active sources set to zero i.e with all the independent voltage sources short-circuited and all the independent current sources open circuited.

Example:- Find V_X by first finding V_{TH} and R_{TH} to the left of A-B



Solution:- step1. First remove everything to the right of A-B.

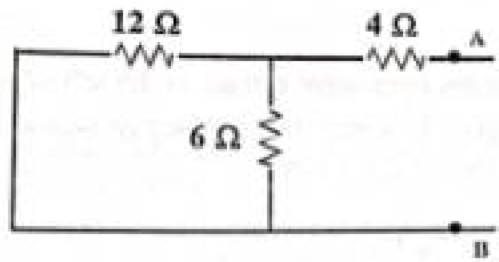


$$V_{AB} = \frac{(30)(6)}{6+12} = 10V$$

Notice that there is no current flowing in the 4 Ω resistor (A-B) is open. Thus there can be no voltage across the resistor.

Step 2. To find R_{th}

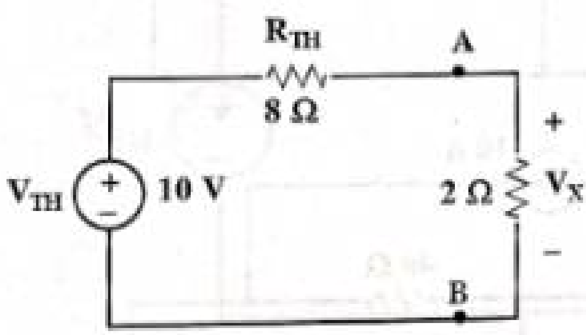
We now deactivate the sources to the left of A-B and find the resistance seen looking in these terminals.



$R_{TH} = 12 \| 6 + 4 = 8\,\Omega$

Step 3. To find V_x

After having found the Thevenin circuit, we connect this to the load in order to find V_X.

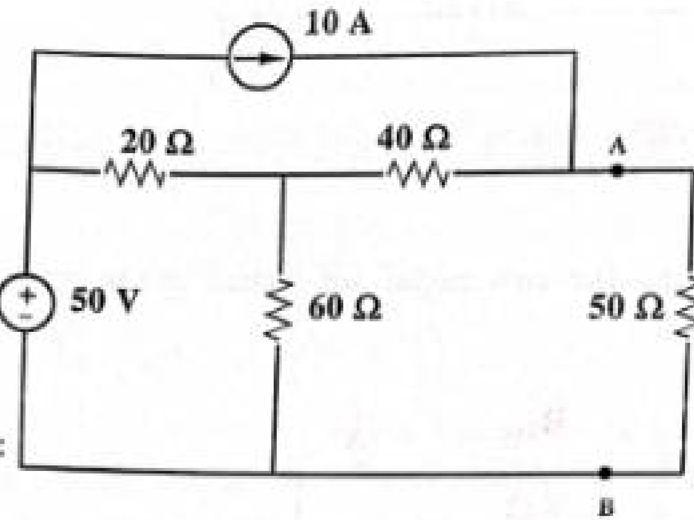


$$V_X = \frac{(10)(2)}{2+8} = 2V$$

4. **NORTON'S THEOREM**:

Statement: Any two terminal linear active network (containing independent voltage and current sources), may be replaced by a constant current source I_N in parallel with a resistance R_N, where I_N is the current flowing through a short circuit placed across the terminals and R_N is the equivalent resistance of the network as seen from the two terminals with all sources replaced by their internal resistance.

Example: Find the Norton equivalent circuit to the left of terminals A-B fo the network shown below. Connect the Norton equivalent circuit to the loa and find the current in the 50 Ω resistor.



Solution:

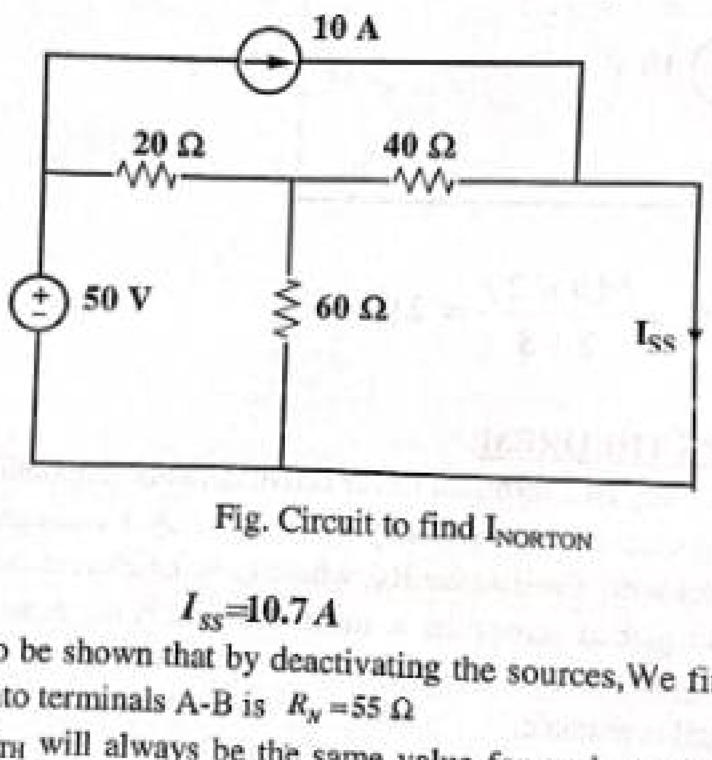
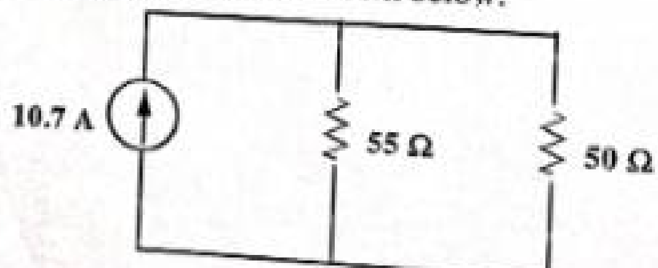


Fig. Circuit to find I_{NORTON}

$$I_{SS}=10.7\,A$$

It can also be shown that by deactivating the sources,We find the resistance looking into terminals A-B is $R_N = 55\ \Omega$

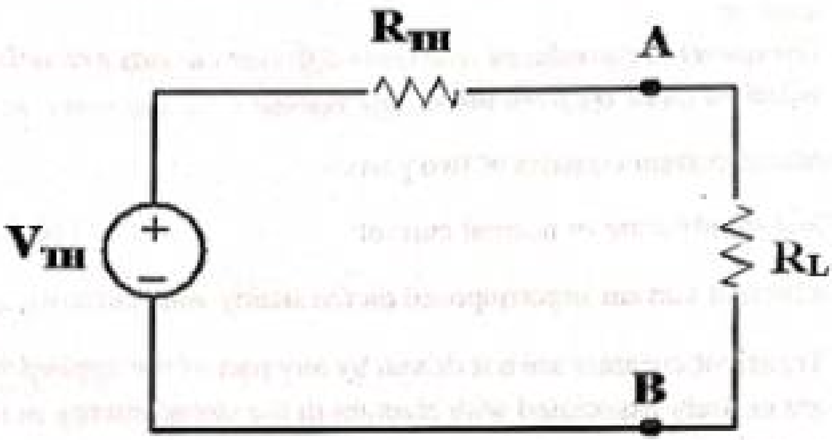
R_N and R_{TH} will always be the same value for a given circuit. The Norton equivalent circuit tied to the load is shown below.



5. **MAXIMUM POWER TRANSFER THEOREM:**

- **Statement:** For any power source, the maximum power transferred from the power source to the load is when the resistance of the load R_L is equal to the equivalent or input resistance of the power source ($R_{in} = R_{Th}$ or R_N).
- The process used to make $R_L = R_{in}$ is called impedance matching.

Explanation:



$$I = \frac{V_{TH}}{R_{TH} + R_L}$$

$$P_L = I^2 R_L = \frac{V_{TH}^2 R_L}{(R_{TH} + R_L)^2}$$

For P_L to be maximum, $\frac{dP_L}{dR_L} = 0$

Or, $R_L = R_{TH}$

So, Maximum power drawn by $R_L = I^2 R_L = \frac{V_{TH}^2 R_L}{(2R_L)^2} = \frac{V_{TH}^2}{4R_L}$

Power supplied by the source$= \frac{V_{TH}^2}{(R_{TH} + R_L)}$

TRANSIENTS

Statement: Sudden change in electrical circuit.

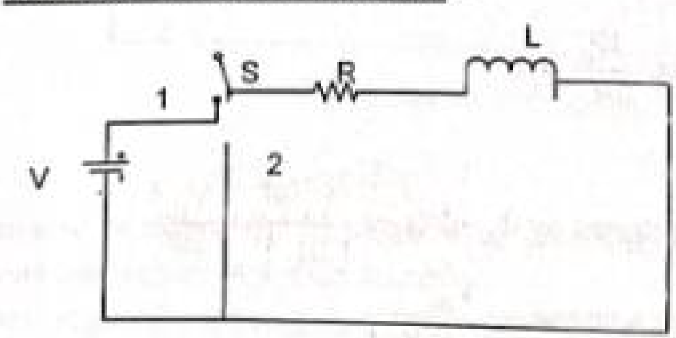
- Amplitude dies out and frequency is more
- Transient disturbances are produced whenever:-
  - An apparatus or circuit is suddenly connected to or disconnected from the supply.
  - A circuit is shorted
  - There is a sudden change in the applied voltage from one finite value to another.
  - *Transients are produced whenever different circuits are suddenly switched on or off from the supply voltage.*

Resultant current consists of two parts:-

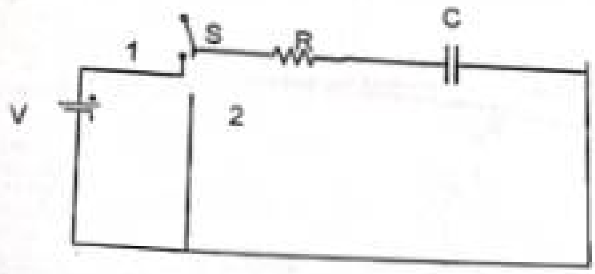
- A final steady state or normal current
- A transient current superimposed on the steady state current

- Transient currents are not driven by any part of the applied voltage but are entirely associated with changes in the stored energy in inductors and capacitors.
- Since there is no stored energy in resistors, there are no transients in pure resistive circuit.

Transient in R-L Series circuit:-



When Switch 'S' is connected to '1',



Charging of RC

$V=V_R+V_C$

When switch is connected to '1' (charging):-

$V=V_R+V_c$

$V=iR+\frac{1}{C}\int i\,dt$

$R\frac{di}{dt}+\frac{i}{C}=0$

$\frac{di}{dt}+\frac{1}{RC}i=0$

$i=Ke^{\frac{-t}{RC}}$

At $t=0^+;\ i=I_0e^{\frac{-t}{\tau}}$

$K=\frac{V}{R};\ So,\ i=\frac{V}{R}e^{\frac{-t}{RC}}$

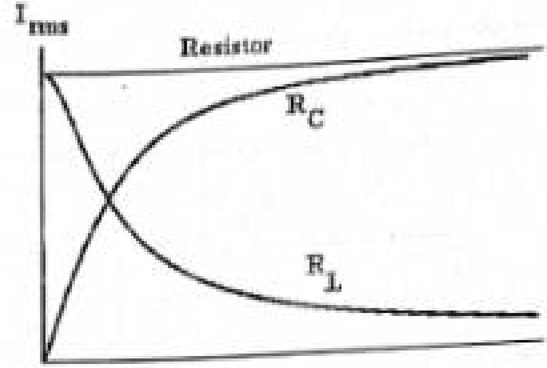
$V_R=iR=I_0e^{\frac{-t}{\tau}}R=Ve^{\frac{-t}{\tau}}$

$V_c=\frac{1}{C}\int i\,dt=\frac{1}{C}\int_0^t I_0e^{\frac{-t}{\tau}}$

$V_c=\frac{1}{C}I_0(-\tau)\left[\frac{-t}{\tau}\right]_0^t=\frac{1}{C}\frac{V}{R}(-RC)$

$V_c=-V\left(e^{\frac{-t}{\tau}}-e^0\right)$

$V_c=V\left(1-e^{\frac{-t}{\tau}}\right)$



<u>Discharging of RC</u>

When connected to '2' in the Fig. above,

$$R\,i + \frac{1}{C}\int i\,dt = 0$$

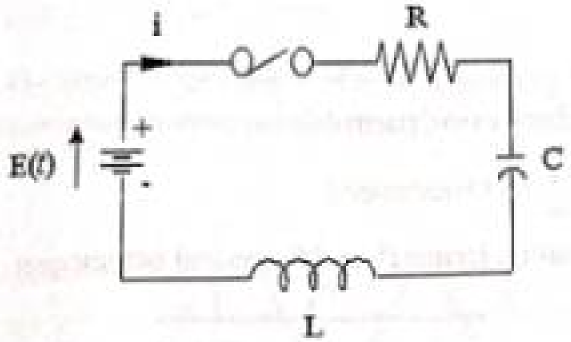
$$R\,\frac{di}{dt} + \frac{i}{C} = 0$$

$$i = K\,e^{\frac{-t}{RC}}$$

At $t = 0; i = \frac{-V}{R}$ (voltage across capacitor starts discharging in opposite direction to the original current dir

$$i = -I_0\,e^{\frac{-t}{RC}} = -I_0\,e^{\frac{-t}{\tau}}$$

Transient in R-L-C Series Circuit



- Two types of energy:- Electromagnetic and electrostatic. So any sudden change in the conditions of the circuit involves redistribution of these two energies.
- Transient current produced due to this redistribution may be unidirectional and decaying oscillatory.

From the above Fig.

$$iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = V$$

$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i}{C} = 0$$

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = 0$$

$$\alpha \pm \beta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\alpha \pm \beta = \frac{\frac{-R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}}{2}$$

$$\alpha \pm \beta = \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$\alpha = \frac{-R}{2L} \quad and\ \beta = \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

Solution of differential equation is:-

$i = K_1 e^{P_1 t} + K_2 e^{P_2 t}$

Roots are: $\alpha + \beta = P_1 ; \alpha - \beta = P_2$

K_1 & K_2 depends on boundary condition

Case 1: High loss circuit:- $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$ i.e Overdamped

In this case, β is positive real quantity. Hence P_1 and P_2 are real but unequ

$$i = K_1 e^{P_1 t} + K_2 e^{P_2 t}$$

$$i = K_1 e^{(\alpha + \beta)t} + K_2 e^{(\alpha - \beta)t}$$

$$i = K_1 e^{\alpha t} e^{\beta t} + K_2 e^{\alpha t} e^{-\beta t}$$

$$i = e^{\alpha t}\left[K_1 e^{\beta t} + K_2 e^{-\beta t}\right]$$

The expression of 'i' is over damped transient non-oscillatory current.

CASE 2:- Low-loss circuit: $\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$ i.e Underdamped

In this case, β is imaginary. Hence roots are complex conjugate

$$P_1 = \alpha + j\beta ; P_2 = \alpha - j\beta$$

$$i = K_1 e^{P_1 t} + K_2 e^{P_2 t}$$

$$i = K_1 e^{(\alpha + j\beta)t} + K_2 e^{(\alpha - j\beta)t}$$

$$i = K_1 e^{\alpha t} e^{j\beta t} + K_2 e^{\alpha t} e^{-j\beta t}$$

$$i = e^{\alpha t}\left[K_1 e^{j\beta t} + K_2 e^{-j\beta t}\right]$$

The expression of 'i' is damped oscillatory

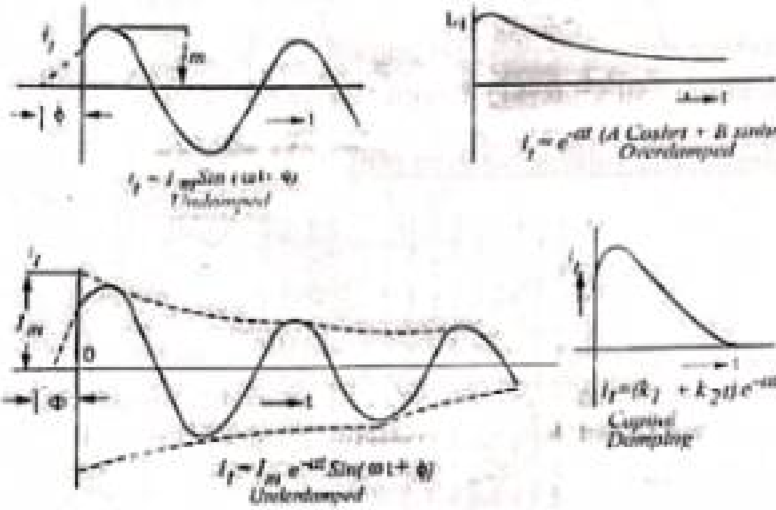
CASE 3: $\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$ i.e Critical damping

In this case β =0, Hence roots P_1 & P_2 are real and equal.

$P_1 = \alpha + 0 = \alpha ; P_2 = \alpha - 0 = \alpha$

$i = K_1 e^{\alpha t} + K_2 t e^{\alpha t}$

The above expression is of critical damping because current is reduced to almost zero in the shortest possible time.



Example: A coil having a resistance of 2Ω and an inductance of 1 H is switched on to a 10 V D.C supply. Write down the expression of current i(t) in the coil as a function of time

Ans: R = 2Ω, L=1 H, V=10 V

Time constant (τ) = L/R=1/2=0.5 sec

Steady current = V/R = 10/2= 5 A

$$i(t) = \frac{V}{R}\left(1 - e^{\frac{-t}{\tau}}\right)$$

$$i(t) = 5\left(1 - e^{\frac{-t}{0.5}}\right) A$$

SINGLE PHASE A.C CIRCUIT

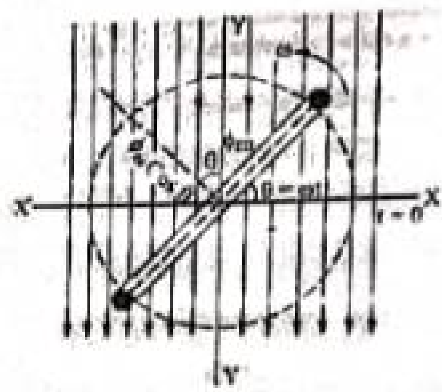
Single phase EMF generation:

Alternating voltage may be generated

1) By rotating a coil in a magnetic field
2) By rotating a magnetic field within a stationary coil

The value of voltage generated depends upon
1) No. of turns in the coil 2) field strength 3) speed

Equation of alternating voltage and current



N= No. of turns in a coil

Φ_m= Maximum flux when coil coincides with X-axis

ω= angular speed (rad/sec) = 2πf

At θ=ωt, Φ= flux component ⊥ to the plane $=\Phi_m \cos \omega t$

According to the Faraday's law of electromagnetic induction,

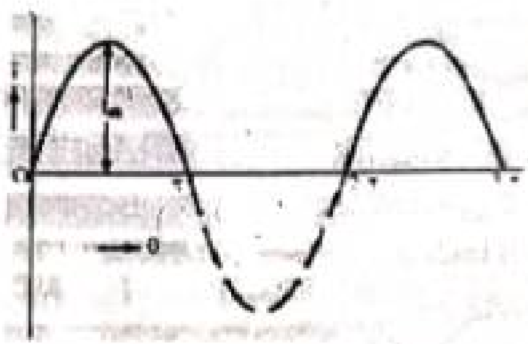
$$e=-N\frac{d\varphi}{dt}=-N\frac{d}{dt}\varphi_m \cos\omega t=\omega N\varphi_m \sin\omega t \quad \text{............(1)}$$

Now, e is maximum value of E_m, when Sinθ = Sin 90° = 1

i.e $E_m = \omega N\Phi_m$ ....(2)

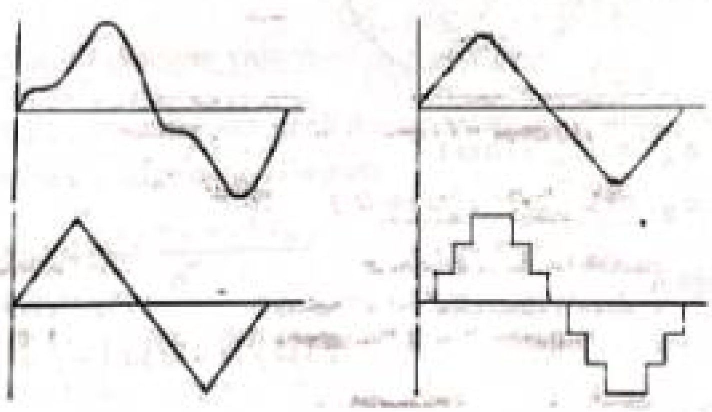
From Eq^n (1) & (2), $e = E_m \sin \omega t$ volt

Now, current (i) at any time in the coil is proportional to the induced emf (e) in the coil. Hence, $i = I_m \sin \omega t$ amp



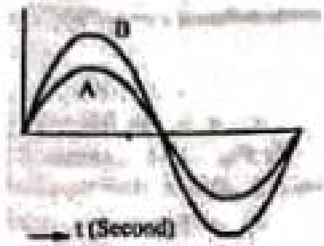
A.C terms:-

- Cycle:- A complete set of positive and negative values of an alternating quantity is known as cycle.



- Time period: The time taken by an alternating quantity to complete one cycle is called time T.
- Frequency: It is the number of cycles that occur in one second. $f = 1/T$
  $f = PN/120$ where, P= No. of poles, N= Speed in rpm
- Waveform: A curve which shows the variation of voltage and current w.r.t time or rotation.

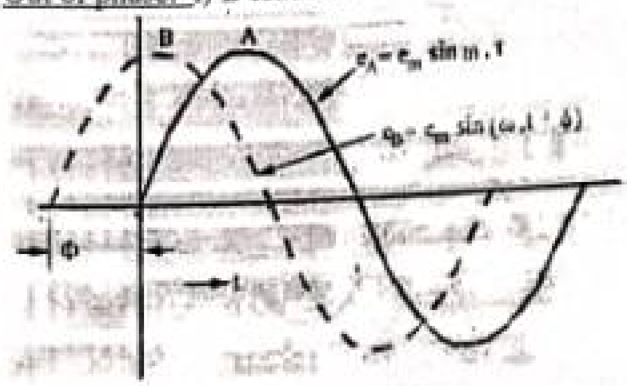
- **Phase & Phase difference:**



<u>In phase:</u>

$$e_A = E_{mA} \sin \omega t$$
$$e_B = E_{mB} \sin \omega t$$

<u>Out of phase:</u> i) B leads A



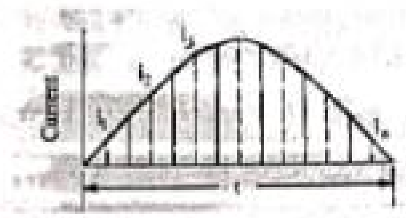
Phase difference Φ.

$$e_A = E_m \sin \omega t$$
$$e_B = E_{mB} \sin(\omega t + \alpha)$$

ii) A leads B or B lags A ,

$$e_A = E_m \sin \omega t$$
$$e_B = E_m \sin(\omega t - \alpha)$$

Root mean Square (RMS) or effective or virtual value of A.C:-



$I_{rms} = \sqrt{\frac{i_1^2 + i_2^2 + \ldots + i_n^2}{n}}$ = Square root of the mean of square of the instantaneous currents

- It is the square root of the average values of square of the alternating quantity over a time period.

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2(\omega t)\,d(\omega t)}$$

Average Value (or mean value):-

- It is the arithmetic sum of all the instantaneous values divided by the number of values used to obtain the sum

$$I_{av} = \frac{i_1 + i_2 + \ldots + i_n}{n}$$

$$I_{av} = \frac{1}{T}\int_0^T i(\omega t)\,d(\omega t)$$

Form factor (K_f):- is the ratio of rms value to average value of an alternating quantity. ($K_f = I_{rms}/I_{av}$)

Peak factor (K_a) or crest factor:- is the ratio of peak (or maximum) value to the rms value of alternating quantity ($Ka = I_{max}/I_{rms}$)

Example: An alternating current varying sinusoidally with a frequency of 50 Hz has an RMS value of 20 A. Write down the equation for the instantaneous value and find this value a) 0.0025 sec b) 0.0125 sec after passing through a positive maximum value. At what time, measured from a positive maximum value, will the instantaneous current be 14.14 A?

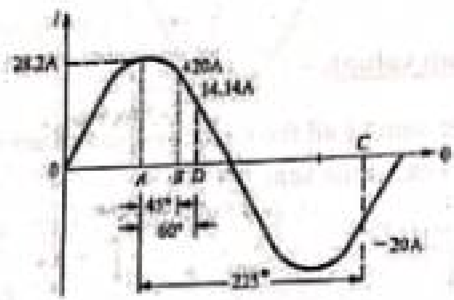
Ans:
$$I_m = 20\sqrt{2} = 28.2\,A$$
$$\omega = 2\pi \times 50 = 100\pi \text{ rad/s}$$

The equation of the sinusoidal current wave with reference to point O as zero time point is

$i = 28.2 \sin 100\pi t$ Ampere

Since time values are given from point A where voltage has positive and maximum value, the equation may itself be referred to point A. In this case, equation becomes

$i = 28.2 \cos 100\pi t$



i) When t= 0.0025 second

$i = 28.2 \cos 100\pi \times 0.0025$ ....angle in radian

$= 28.2 \cos 100 \times 180 \times 0.0025$ ....angle in degrees

$= 28.2 \cos 45° = 20$ A ....point B

ii) When t = 0.0125 sec

$I = 28.2 \cos 100 \times 180 \times 0.0125$

$= 28.2 \cos 225° = 28.2 \times (-1/\sqrt{2})$

$= -20$ A ....point C

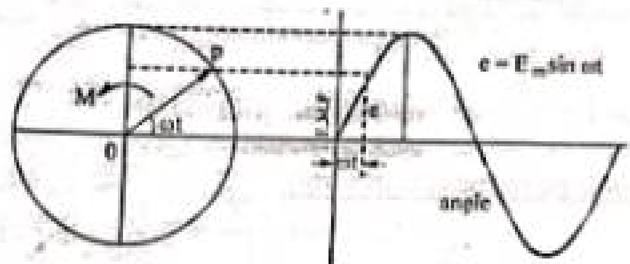
iii) Here $i = 14.14$ A

$14.14 = 28.2 \cos 100 \times 180\, t$

$\cos 100 \times 180\, t = ½$

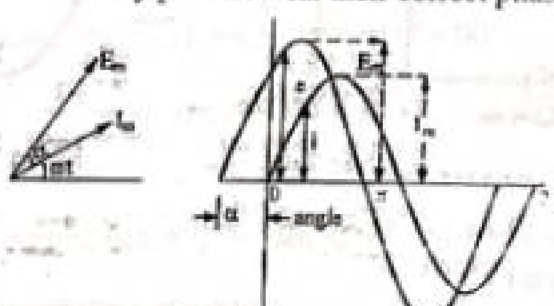
Or, $100 \times 180\, t = \cos^{-1}(1/2) = 60°$, $t = 1/300$ sec ....point D

Phasor & Phasor diagram:



Phasor: Alternating quantities are vector (i.e having both magnitude and direction). Their instantaneous values are continuously changing so that they are represented by a rotating vector (or phasor). A phasor is a vector rotating at a constant angular velocity

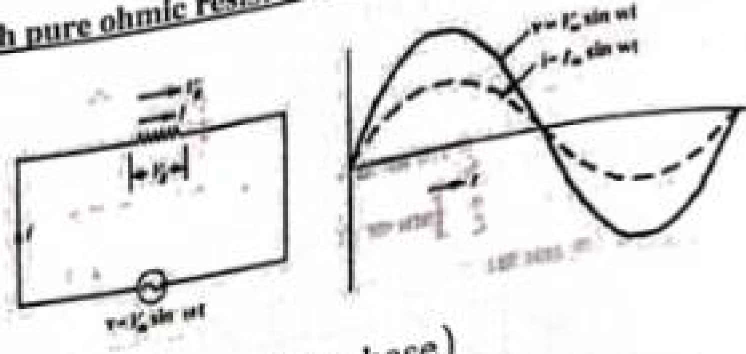
Phasor diagram: is one in which different alternating quantities of the same frequency are represented by phasors with their correct phase relationship



Points to remember:

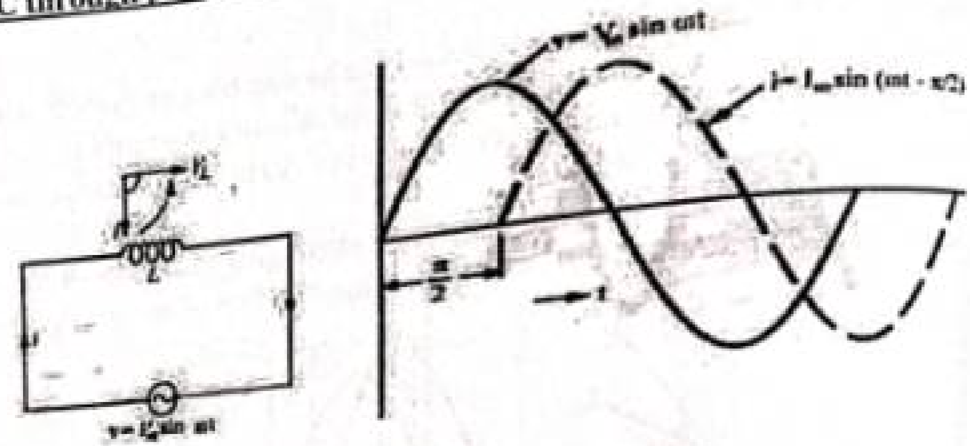
1. The angle between two phasors is the phase difference
2. Reference phasor is drawn horizontally
3. Phasors are drawn to represent rms values
4. Phasors are assumed to rotate in anticlockwise direction
5. Phasor diagram represents a "still position" of the phasors in one particular point

A.C through pure ohmic resistance only



$$v = iR \text{ or } i = \frac{v}{R} = \frac{V_m}{R}\sin\omega t \text{ (in phase)}$$

A.C through pure inductance only



$$v = L\frac{di}{dt} = V_m \sin\omega t$$

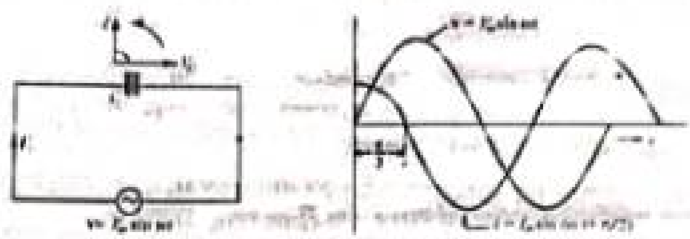
$$i = \frac{V_m}{L}\int \sin\omega t$$

$$i = -\frac{V_m}{\omega L}\cos\omega t$$

$$i = I_m \sin\left(\omega t - \frac{\pi}{2}\right) \quad \text{(current lags by } 90^\circ\text{)}$$

$$\omega L = 2\pi f L = X_L = \text{inductive reactance (in } \Omega\text{)}$$

A.C through pure Capacitance only



$$i = C\frac{dv}{dt} = C\frac{d}{dt}(V_m \sin\omega t)$$
$$= \omega C V_m \cos\omega t$$
$$= \omega C V_m \sin\left(\omega t + \frac{\pi}{2}\right) = \frac{V_m}{\frac{1}{\omega C}}\sin\left(\omega t + \frac{\pi}{2}\right)$$
$$= I_m \sin\left(\omega t + \frac{\pi}{2}\right) \quad \text{(current leads by } 90^\circ\text{)}$$
$$\frac{1}{\omega C} = X_C = \frac{1}{2\pi f C} = \text{capacitive reactance (in } \Omega\text{)}$$

'j' operator: j is a operator which rotates a vector by 90° in anticlockwise direction

$j^2 = -1\,; j = \sqrt{-1}$

Note: 'i' is used for current hence 'j' is used to avoid confusion

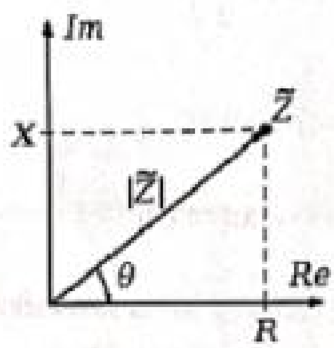
Mathematical representation of vectors:

1. Rectangular or Cartesian form :- $\vec{V} = a \pm jb$
2. Polar form : $\vec{V} = V\angle\pm\theta$
3. Trignometrical form : $\vec{V} = V(\cos\theta \pm j\sin\theta)$
4. Exponential form : $\vec{V} = Ve^{\pm j\theta}$

Note: rectangular form is best suited for addition and subtraction & polar form is best suited for multiplication and division

IMPEDANCE:

In quantitative terms, it is the complex ratio of the voltage to the current in an alternating current (AC) circuit. Impedance extends the concept of resistance to AC circuits, and possesses both magnitude and phase, unlike resistance, which has only magnitude. When a circuit is driven with direct current (DC), there is no distinction between impedance and resistance; the latter can be thought of as impedance with zero phase angle.



Where X=Total reactance of the network (Both inductive and capacitive)

R=Resistance of the network in ohm.

θ= Phasor angle in degree/Radian.

Note:

I. If θ =0 degree then the load is purely**Resistive.**
II. If θ=-90 degree then the load is purely **inductive.**(lagging)
III. If θ=90 degree then the load is purely **capacitive.**(leading)

Z=R+jX

Where Z=impedance of the electrical network in ohm.

R=Resistance of the network in ohm.

X=Reactance of the electrical network in ohm.

Admittance:

In electrical engineering, admittance is a measure of how easily a circuit or device will allow a current to flow. It is defined as the inverse of impedance. The SI unit of admittance is the siemens (symbol S).

Admittance is defined as:

$$Y = 1/Z$$

Where

Y is the admittance, measured in siemens

Z is the impedance, measured in ohms

The synonymous unit mho, and the symbol ℧ (an upside-down uppercase omega Ω), are also in common use.

Resistance is a measure of the opposition of a circuit to the flow of a steady current, while impedance takes into account not only the resistance but also dynamic effects (known as reactance). Likewise, admittance is not only a measure of the ease with which a steady current can flow, but also the dynamic effects of the material's susceptance to polarization:

$$Y = G + j\,B$$

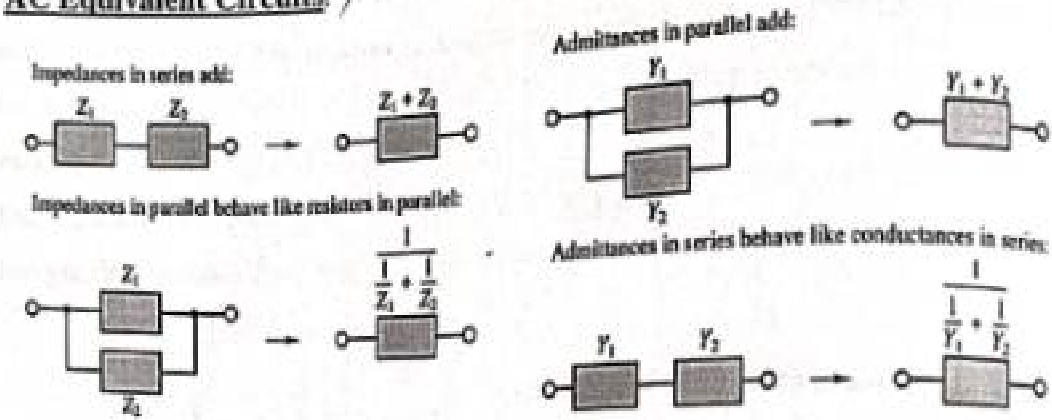
Where

Y is the admittance, measured in siemens.

G is the conductance, measured in siemens.

B is the susceptance, measured in siemens.

AC Equivalent Circuits:



1. Impedances in series add together to give the equivalent impedance while the admittance in parallel add together to give the equivalent admittance.
2. Impedances in parallel gives equivalent impedance by reciprocating the reciprocal sum of the impedances and to obtain the equivalent admittance in series same procedure has to be followed.

Instantaneous and Average Power

The most general expressions for the voltage and current delivered to an arbitrary load are as follows:

$v(t) = V\cos(\omega t - \theta_V)$

$i(t) = I\cos(\omega t - \theta_I)$

Since the instantaneous power dissipated by a circuit element is given by the product of the instantaneous voltage and current, it is possible to obtain a general expression for the power dissipated by an AC circuit element:

$p(t) = v(t)i(t) = V\, I \cos(\omega t) \cos(\omega t - \theta)$

It can be further simplified with the aid of trigonometric identities to yield

$p(t) = V\, I/2 \cos(\theta) + V\, I/2 \cos(2\omega t - \theta)$

where θ is the difference in phase between voltage and current

The average power corresponding to the voltage and current signal can be obtained by integrating the instantaneous power over one cycle of the sinusoidal signal. Let $T = 2\pi/\omega$ represent one cycle of the sinusoidal signals. Then the average power, Pav, is given by the integral of the instantaneous power,

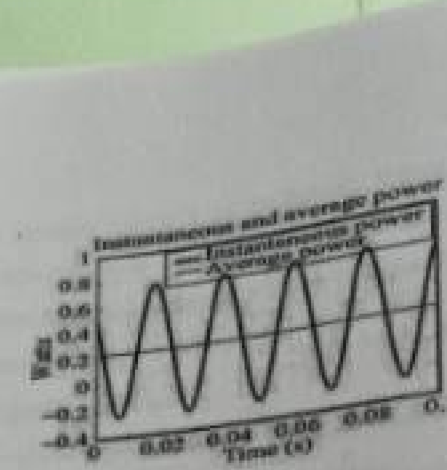
p(t), over one cycle:

$$P_{av} = \frac{1}{T}\int_0^T p(t)\, dt$$

$$= \frac{1}{T}\int_0^T \frac{VI}{2}\cos(\theta)\, dt + \frac{1}{T}\int_0^T \frac{VI}{2}\cos(2\omega t - \theta)\, dt$$

$$P_{av} = \frac{VI}{2}\cos(\theta) \qquad \text{Average power}$$

since the second integral is equal to zero and $\cos(\theta)$ is a constant.



In phasor notation, the current and voltage are given by

$$V(j\omega) = Ve^{j0}$$
$$I(j\omega) = Ie^{-j\theta}$$

impedance of the circuit elementdefined by the phasor voltage and currentto be

$$Z = \frac{V}{I}e^{-j(\theta)} = |Z|e^{j\theta_z}$$

The expression for the average power using phasor notation

$$P_{av} = \frac{1}{2}\frac{V^2}{|Z|}\cos\theta = \frac{1}{2}I^2|Z|\cos\theta$$

Power Factor

The phase angle of the load impedance plays a very important role in the absorption of power by load impedance. The average power dissipated by an AC load is dependent on the cosine of the angle of the impedance. To recognize the importance of this factor in AC power computations, the term cos(θ) is referred to as the power factor (pf). Note that the power factor is equal to 0 for a purely inductive or capacitive load and equal to 1 for a purely resistive load; in every other case, $0<pf<1$. If the load has an inductive reactance, then θ is positive and the current lags (or follows) the voltage. Thus, when θ and Q are positive, the corresponding power factor is termed lagging. Conversely, a capacitive load will have a negative Q, and hence a negative θ. This corresponds to a leading power factor, meaning that the load current leads the load voltage. A power factor close to unity signifies an efficient transfer of energy from the AC source to the load, while a small power factor corresponds to inefficient use of energy. Two equivalent expressions for the power factor are given in the following:

$$pf = \cos(\theta) = \frac{P_{av}}{\tilde{V}\tilde{I}} \quad \text{Power factor}$$

where $\tilde{V}$ and $\tilde{I}$ are the rms values of the load voltage and current.

Complex Power

The expression for the instantaneous power may be further expanded to provide further insight into AC power. Using trigonometric identities, we obtain the following expressions:

$$\begin{aligned} p(t) &= \frac{\tilde{V}^2}{|Z|}[\cos\theta + \cos\theta\cos(2\omega t) + \sin\theta\sin(2\omega t)] \\ &= \tilde{I}^2|Z|[\cos\theta + \cos\theta\cos(2\omega t) + \sin\theta\sin(2\omega t)] \\ &= \tilde{I}^2|Z|\cos\theta(1+\cos(2\omega t)) + \tilde{I}^2|Z|\sin\theta\sin(2\omega t) \end{aligned}$$

Recalling the geometric interpretation of the impedance Z

$|Z|\cos\theta = R$ and $|Z|\sin\theta = X$

are the resistive and reactive components of the load impedance, respectively. On the basis of this fact, it becomes possible to write the instantaneous power as:

$$p(t) = \tilde{I}^2 R(1 + \cos(2\omega t)) + \tilde{I}^2 X \sin(2\omega t)$$
$$= \tilde{I}^2 R + \tilde{I}^2 R \cos(2\omega t) + \tilde{I}^2 X \sin(2\omega t)$$

Since Pav corresponds to the power absorbed by the load resistance, it is also called the real power, measured in units of watts (W). On the other hand, Q takes the name of reactive power, since it is associated with the load reactance. The units of Q are volt-amperes reactive, or VAR. Note that Q represents an exchange of energy between the source and the reactive part of the load; thus, no net power is gained or lost in the process, since the average reactive power is zero. In general, it is desirable to minimize the reactive power in a load.

The computation of AC power is greatly simplified by defining a fictitious but very useful quantity called the complex power, S:

$$S = \tilde{V}\tilde{I}^*$$

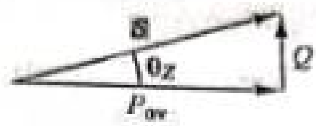
where the asterisk denotes the complex conjugate You may easily verify that this definition leads to the convenient expression

$$S = \tilde{V}\tilde{I}\cos\theta + j\tilde{V}\tilde{I}\sin\theta = \tilde{I}^2 R + j\tilde{I}^2 X = \tilde{I}^2 Z$$

or

$$S = P_{av} + jQ$$

The complex power S may be interpreted graphically as a vector in the complex S plane



$$|S| = \sqrt{P_{av}^2 + Q^2} = \tilde{V} \cdot \tilde{I}$$

$$P_{av} = \tilde{V}\tilde{I} \cos \theta$$

$$Q = \tilde{V}\tilde{I} \sin \theta$$

The magnitude of S, |S|, is measured in units of volt-amperes (VA) and is called apparent power, because this is the quantity one would compute by measuring the rms load voltage and currents without regard for the phase angle of the load.The complex power may also be expressed by the product of the square of the rms current through the load and the complex load impedance:

$$S = \tilde{I}^2 Z$$

or

$$\tilde{I}^2 R + j\tilde{I}^2 X = \tilde{I}^2 Z$$

or, equivalently, by the ratio of the square of the rms voltage across the load to the complex conjugate of the load impedance:

$$S = \frac{\tilde{V}^2}{Z^*}$$

Active, Reactive and Apparent Power

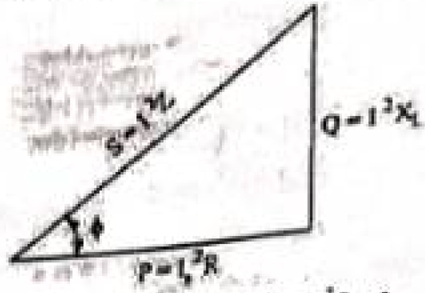


Fig. Power Triangle

$S^2 = P^2 + Q^2$

$S = P + jQ$

- **Apparent power, S**: is the product of rms values of the applied voltage and circuit current. It is also known as wattless (idle) component
  $S=VI=IZ\times I=I^2Z$ volt-amp
- **Active power or true power, P**: is the power which actually dissipated in the circuit resistance. It is also known as wattful component of power.
  $P=I^2R=I^2Z\cos\Phi=VI\cos\Phi$ watt
- **Reactive power, Q**:- is the power developed in the reactance of the circuit.
  $Q=I^2X=I^2Z\sin\Phi=VI\sin\Phi$ VAR

Example: In a particular R-L series circuit a voltage of 10 V at 50 Hz produces a current of 700 Ma while the same voltage at 75 Hz produces 500 mA. What are the values of R and L in the circuit.

Ans. i)

$Z=\sqrt{R^2+(2\pi\times 50L)^2}=\sqrt{R^2+98696L^2}$

$V=IZ$ or $10=700\times 10^{-3}\sqrt{(R^2+98696L^2)}$

$\sqrt{(R^2+98696L^2)}=10/700\times 10^{-3}=100/7$

or $R^2+98696L^2=10000/49$..............(i)

ii) In the second case

$$Z=\sqrt{R^2+(2\pi\times 75L)^2}=\sqrt{R^2+(222066L^2)}$$

$$10=500\times 10^{-3}\sqrt{(R^2+222066L^2)}$$

$$\sqrt{(R^2+222066L^2)}=20$$

$$R^2+222066L^2=400 \quad \text{.................(ii)}$$

subtracting eq(i) from eq(ii), we get

$$222066L^2-98696L^2=400-(10000/49)$$

$$123370L^2=196$$

$$L=0.0398\ H=40mH$$

Substituting this value of L in eq(ii),we get

$$R^2+222066(0.398)^2=400$$

$$R=6.9\ \Omega$$

Introduction to resonance in series & parallel circuit

Resonance:

Definition: An AC circuit is said to be in resonance when the circuit current is in phase with the applied voltage. So, the power factor of the circuit becomes unity at resonance and the impedance of the circuit consists of only resistance.

Series Resonance: In R-L-C series circuit, both X_L and X_C are frequency dependent. If we vary the supply frequency then the values of X_L and X_C varies. At a certain frequency called resonant frequency (f_r), X_L becomes equal to X_C and series resonance occurs.

At series resonance, $X_L = X_C$

$2\pi f_r L = 1/2\pi f_r C$

$f_r = 1/2\pi\sqrt{LC}$

Impedance of RLC series circuit is given by:

$$Z=\sqrt{R^2+(X_L-X_C)^2} \quad (\text{Since}, X_L=X_C)$$

$$Z=\sqrt{R^2}$$

$$Z=R$$

$$\cos\varphi=\frac{R}{Z}=\frac{R}{R}=1$$

Properties of series resonance:-

In series resonance,

- The circuit impedance Z is minimum and equal to the circuit resistance R.
- The circuit current I= V/Z = V/R and the current is maximum
- The power dissipated is maximum, $P=V^2/R$
- Resonant frequency is fr = 1/2π√LC
- Voltage across inductor is equal and opposite to the voltage across capacitor
- Since power factor is 1, so zero phase difference. Circuit behaves as a purely resistive circuit.

$$V = R\,i + L\,\frac{di}{dt}$$

$$i = i_{ss} + i_{tr}$$

$$i_{ss} = \frac{V}{R}$$

$$i_{tr} = R\,i + L\,\frac{di}{dt} = 0$$

$$\frac{di}{dt} + \frac{R}{L}i = 0$$

$$\frac{di}{dt} = \frac{-R}{L}i$$

$$\frac{di}{i} = \frac{-R}{L}dt$$

$$\ln i = \frac{-R}{L}t; i_{tr} = K e^{\frac{-R}{L}t}$$

$$i_{ss} = \frac{V}{R}$$

$$i_{tr} = K e^{\frac{-R}{L}t}$$

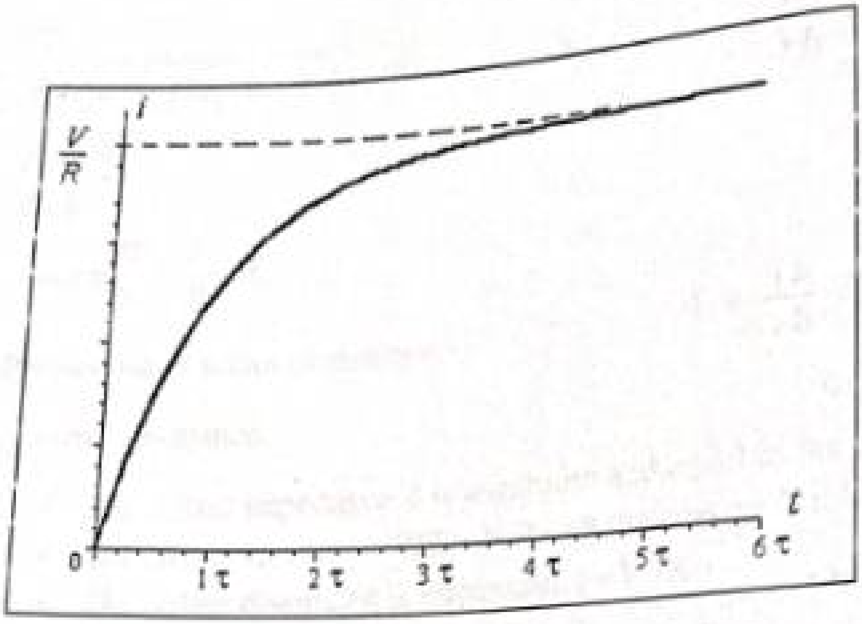
$$i = \frac{V}{R} + K e^{\frac{-R}{L}t}$$

At t=0, i=0 So,

$$0 = \frac{V}{R} + K$$

$$K = \frac{-V}{R}$$

$$i = \frac{V}{R}\left(1 - e^{\frac{-R}{L}t}\right)$$



$\lambda=\frac{L}{R}$ is called time constant and $\frac{R}{L}$ is called damping coefficient of the circuit

$$V_R = iR = V\left(1-e^{-\frac{t}{\lambda}}\right)$$

Emf of self inductance is $-L\frac{di}{dt}=i_t R$

If $t=\lambda$, then $i_t = I_0 e^{-1} = I_0 e^{-1} = I_0/e = I_0/2.718 = 0.37\ I_0$

Hence, time period of a circuit is the time during which the transient current decrease to 0.37 of its initial value.

Transient in R-C Series Circuit:

Consider an ac circuit containing a resistor of resistance R ohms and a capacitor of capacitance C farad across an a.c source of rms voltage V volts as shown in Fig. below:-

Example: A series RLC circuit having a resistance of 50Ω, an inductance of 500 mH and a capacitance of 400 μF, is energized from a 50 Hz, 230 V, AC supply. Find a) resonant frequency of the circuit b) peak current drawn by the circuit at 50 Hz and c) peak current drawn by the circuit at resonant frequency

Ans.

$$a)\ f_0=\frac{1}{2\pi\sqrt{LC}}=\frac{1}{2\pi\sqrt{500\times10^{-3}\times400\times10^{-6}}}=11.25\ Hz$$

b) $R=50\Omega$

$$X_L=\omega L=2\pi\times50\times500\times10^{-3}=157\Omega$$

$$X_C=\frac{1}{\omega C}=\frac{1}{2\pi\times50\times400\times10^{-6}}=7.9\Omega$$

$$X=X_L-X_C=157-7.9=149.1\Omega$$

$$Z=\sqrt{R^2+X^2}=\sqrt{50^2+149.1^2}=157.26\Omega$$

Peak supply voltage, $V_m=\sqrt{2}\ V_{rms}=\sqrt{2}\ (230)=325.26\ V$

Hence peak current at 50Hz $I_m=\frac{V_m}{Z}=\frac{325.26}{157.26}=2.068$

c) At resonance, $Z_0=R=50\Omega$

So, peak current during resonance, $I_{mo}=\frac{V_m}{R}=\frac{325.26}{50}=6.5025A$

Parallel resonance:

Points to remember:

- Net susceptance is zero, i.e $1/X_C=X_L/Z^2$

$X_L \times X_C=Z^2$

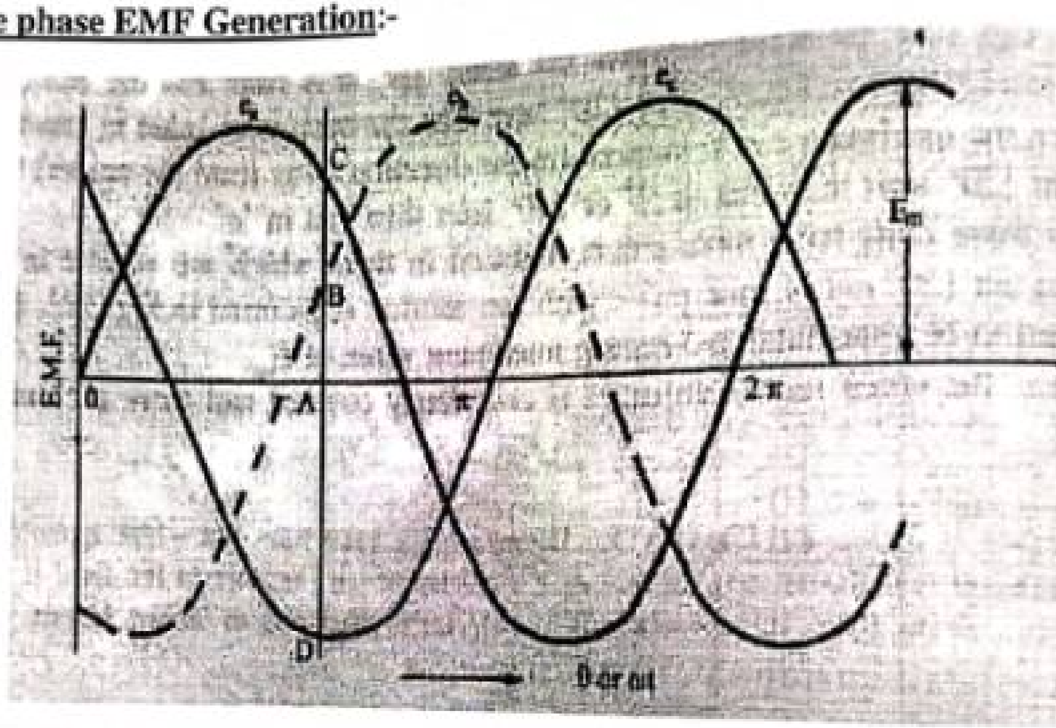
Or $L/C=Z^2$

- The admittance equals conductance
- Reactive or wattless component of line current is zero
- Dynamic impedance = L/CR Ω

- Line current at resonance is minimum and V/L/CR but is in phase with the applied voltage
- Power factor of the circuit is unity

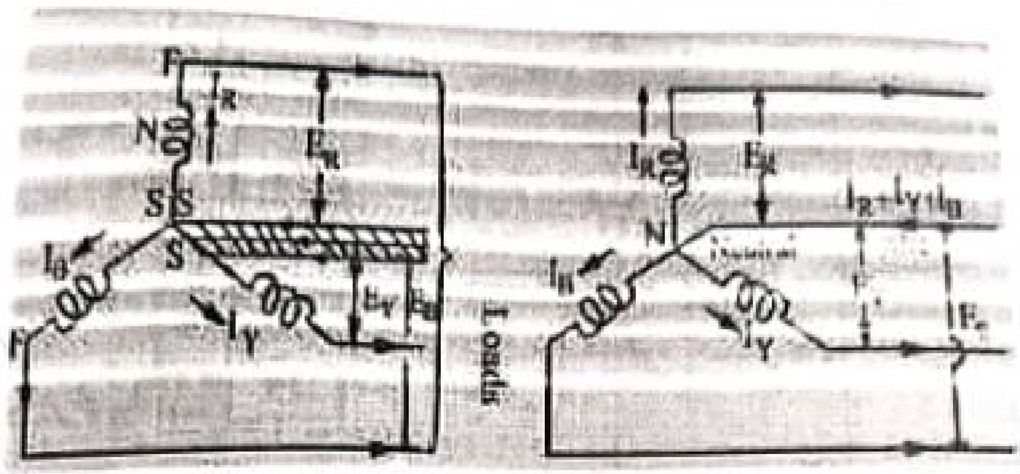
THREE PHASE AC CIRCUIT

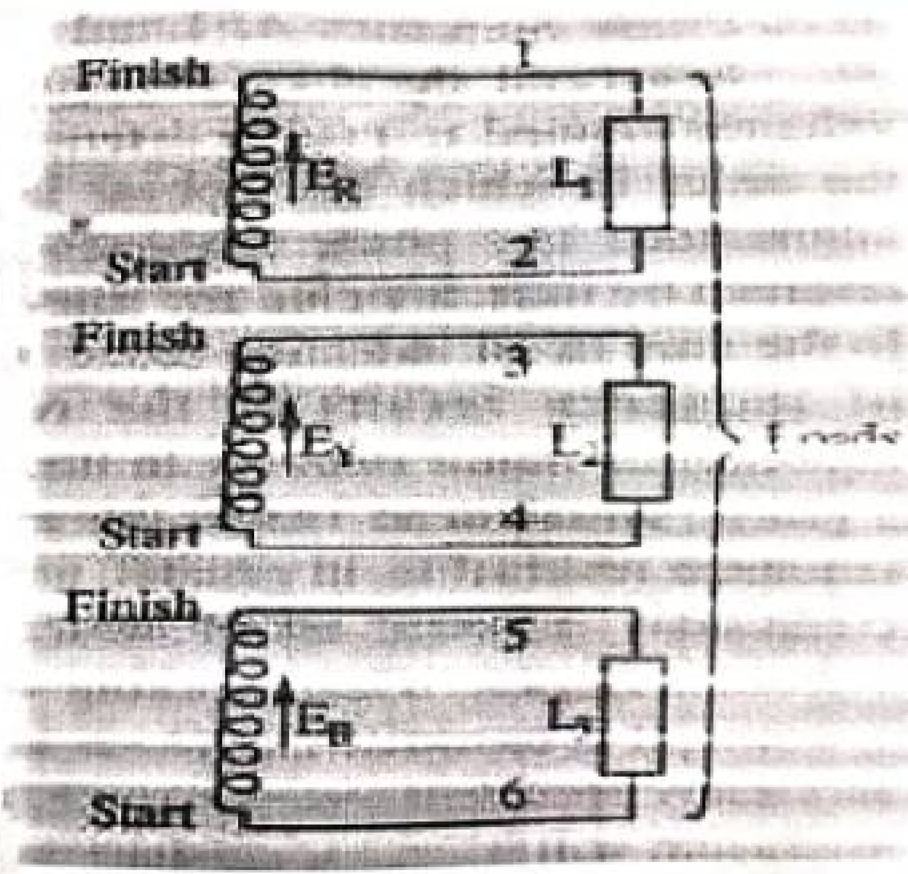
Three phase EMF Generation:-

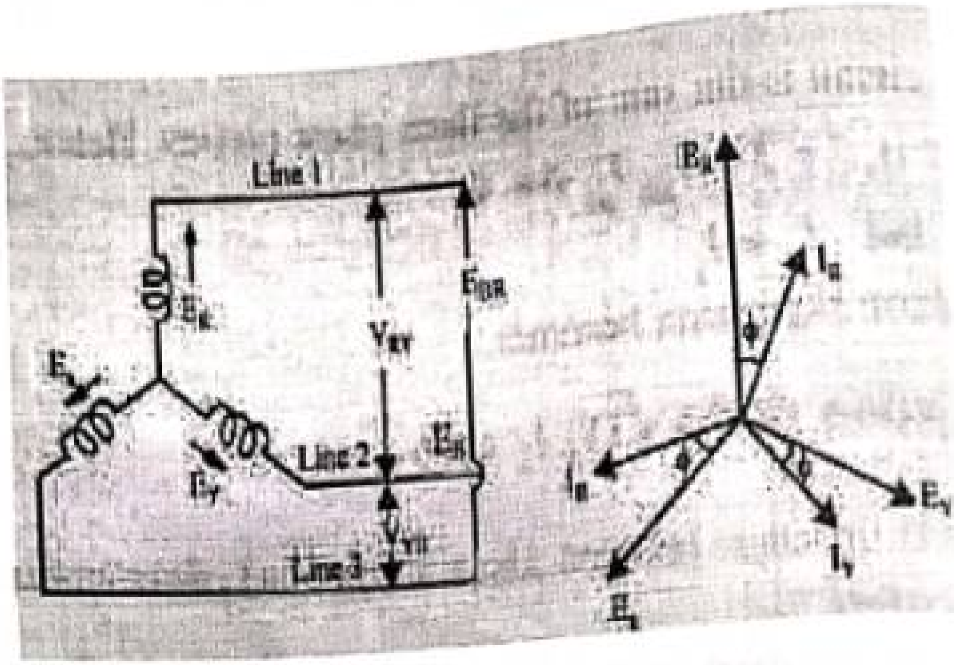


If the 3-coil windings W_1, W_2 and W_3 arranged at 120° apart from each other on the same axis are rotated, then the emf induced in each of them will have a phase difference of 120°. In other words if the emf (or current) in one winding (w_1) has a phase of 0°, then the second winding (w_2) has a phase of 120° and the third (w_3) has a phase of 240°.

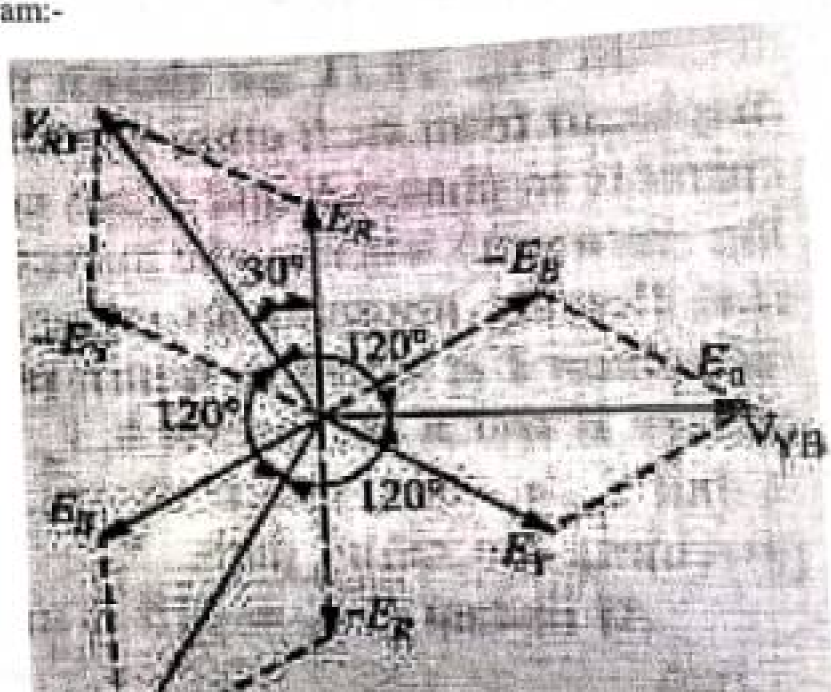
Star (Y) connection:-







Phasor diagram:-



Here, E_R, E_Y, E_B are phase voltages and V_{RY}, V_{YB}, V_{BR} are line voltages

$$V_{RY} = \sqrt{E_R^{\ 2} + E_Y^{\ 2} + 2E_R E_Y \cos 60^\circ}$$

$$= \sqrt{E_R^{\ 2} + E_R^{\ 2} + 2E_R E_R \cos 60^\circ}$$

$$= \sqrt{3}\, E_R$$

Hence.

- Line voltage $= \sqrt{3}$ x phase voltage
- Line current = phase current
- Line voltages are also 120° apart
- Line voltage are 30° ahead of respective phase voltages
- The angle between line voltage and line current is (30°+Φ)

Power: Total power = 3 x phase power

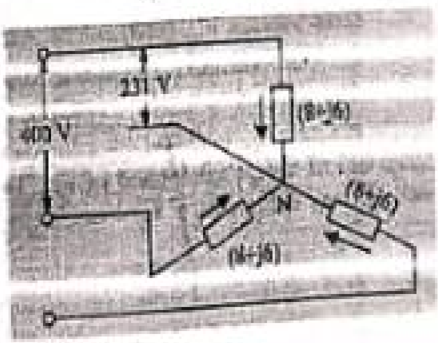
$= 3 \times V_{ph} \times I_{ph} \times \cos\Phi$

$= \sqrt{3}\, V_L\, I_L \cos\Phi$

Φ is the angle between phase voltage and current

Example: A balanced star connected load of (8+j6)Ω per phase is connected to a balanced 3-phase 400 V supply. Find the line current, power factor, power and total volt-amperes.

Ans.



$Z_{ph} = \sqrt{8^2 + 6^2} = 10\Omega$

$V_{ph} = \frac{400}{\sqrt{3}} = 231\,V$

$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{231}{10} = 23.1\text{ A}$

(i) $I_L = I_{ph} = 23.1$ A

(ii) p.f $= \cos\Phi = \frac{R_{ph}}{Z_{ph}} = \frac{8}{10} = 0.8$(lag)

(iii) Power $P = \sqrt{3}\, V_L I_L \cos\Phi$

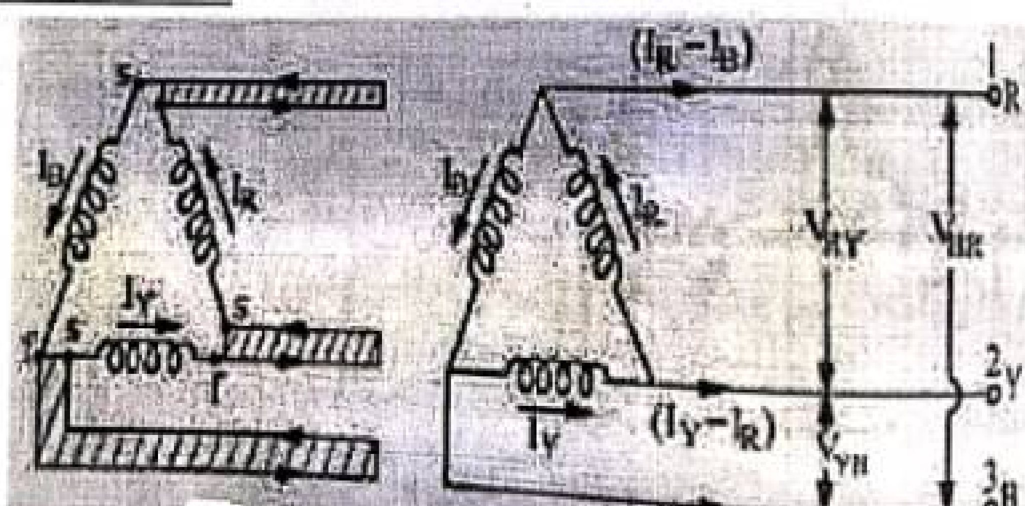
$= \sqrt{3} \times 400 \times 23.1 \times 0.8$

$= 12{,}800$ W [Also, $P = 3I_{ph}^2 R_{ph} = 3(23.1)^2 \times 8 = 12{,}800$ W]

(iv) Total volt-amperes,

$S = \sqrt{3}\, V_L I_L = \sqrt{3} \times 400 \times 23.1 = 16{,}000$ VA

<u>**Delta-connection**</u>:



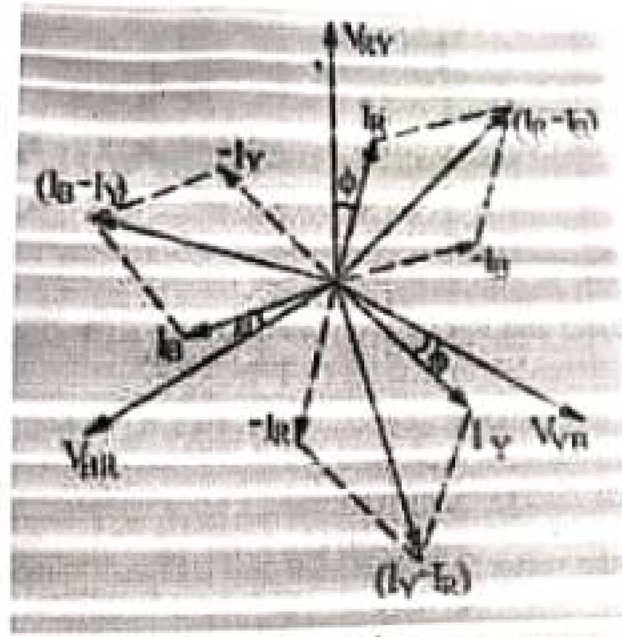


Fig. Phasor Diagram

$$I_L = I_R - I_B$$

$$I_L = \sqrt{I_R^{\,2} + I_B^{\,2} + 2I_R I_B \cos 60^\circ} = \sqrt{I_R^{\,2} + I_R^{\,2} + 2I_R I_R \cos 60^\circ} = \sqrt{3} I_R$$

Hence,

- Line current =√3 phase current
- Line voltage = phase voltage
- Line currents are also 120° apart
- Line currents are 30° behind the respective phase currents
- Angle between line current and line voltage is 30°+Φ

Power: Total power = 3 x phase power

$= 3 \times V_{ph} I_{ph} \cos\Phi$

$= 3 \times V_L \times I_L/\sqrt{3} \times \cos\Phi$

$= \sqrt{3}\, V_L I_L \cos\Phi$

Note: For both star and delta system:

Active & True power = $\sqrt{3}\ V_L\ I_L \cos\Phi$

Reactive power = $\sqrt{3}\ V_L\ I_L \sin\Phi$

Apparent power = $\sqrt{3}\ V_L\ I_L$